3

The time value of money, the dividend discount model and dividend policy

3.1 The time value of money

Ask your friends how much money they would need to be offered in one year’s time to make them indifferent between that amount and a guaranteed £1,000 now. The answer will vary from individual to individual, and will also vary geographically.

Any rational individual will prefer certain money now rather than uncertain money in the future. This is known as the “time value of money”. Even though you do not realise it, when answering the question above, you are implicitly thinking about important concepts such as opportunity cost, inflation and risk. For example, by how much are prices rising, how much satisfaction you would gain from spending the money now and what is the likelihood of the future payment not occurring? In order to determine the future amount that will make us indifferent, an investor will require compensation for these three elements.

The eagerness to spend the £1,000 now is the most difficult to quantify, but for inflation and risk it is easy to assign numerical values. Assume that inflation is 5% p.a. and we attach an amount of 3% to the eagerness to consume. In order to reward an investor for these two factors the required return would need to be:
(1 + 0.05) x (1 + 0.03) – 1 = 8.15%

The closest investment we have that simply rewards investors for loss of consumption and inflation, with no risk, is a short-term government security. In Chapter 2 we referred to such an asset as the risk-free rate.

However, physical investments such as building an office block or manufacturing a new product, and financial investments such as buying shares or bonds, carry with them a degree of risk. Investors therefore demand a “risk premium” beyond the risk-free rate when making investments with uncertain outcomes. We have already seen in section 2.7 that when determining the expected return on an asset we use the security market line:

\[ E(R_i) = R_f + \beta_i [E(R_m) - R_f] \]

More generally we can write:

required return = risk-free rate + risk premium

Therefore, when we are attempting to compare different amounts of money at different points in time, we need to take into consideration the eagerness to spend, inflation and risk.

3.2 Present values

Many topics in finance, including investment appraisal/ NPV analysis, bond pricing and the dividend discount model, to take three examples, have the time value of money at their very heart.

In order to compare like with like, it is important to value cash flows at the same point in time, which might be the current time period or some time period in the future. In order to measure cash flows at a particular point in time we use a tool called “discounted cash flow”. In section 3.1, we assumed a risk-free rate of 8.15%. If the risk premium was a further 10%, then the future value of £1,000 could be calculated as follows:

\[ £1000 \times (1 + 0.0815)^t = £1,081.5 \]

Given these assumptions, a rational investor would be indifferent between £1,000 now and £1,081.50 in the future. Here, we have found the future value of an amount P (here £1,000) at a discount rate of r, in t years’ time.

future value = P x (1 + r)^t

It is more common to consider a future cash flow, CF_t (e.g. a dividend payment or a coupon from a bond), and find its present value:
The time value of money, the dividend discount model and dividend policy

\[
\text{Present Value} = \frac{CF_t}{(1 + r)^t}
\]

Here, we are said to have taken some future cash flow and discounted it back to time zero.

More formally, in order to find the present value of some future cash flow, we multiply the cash flow by the discount factor:

\[
DF = \frac{1}{(1 + r)^t}
\]

\[
PV = DF \times C_t = \frac{C_t}{(1 + r)^t}
\]

where \(C_t\) is the cash flow received in time period \(t\), and \(r\) is the required return/time value of money.

Present values (PV) can be added together to evaluate multiple cash flows:

\[
PV = \frac{C_1}{(1 + r)^1} + \frac{C_2}{(1 + r)^2} + \frac{C_3}{(1 + r)^3} + \ldots
\]

Each individual cash flow is measured in “year zero” money and therefore can be added together, which is the additive property of discount cash flow. It is evident from the equation above that the greater \(r\) is, the higher the denominator, hence the lower the discount factor and consequently the lower the PV. In addition, the further into the future is the cash flow, the higher the denominator, the lower the discount factor and the lower the PV.

Example

i. What is the PV of a cash flow of $100 received in one year’s time at discount rate of 5%?

\[
(i) \quad PV = \frac{$100}{(1+0.05)^1} = $95.24
\]

ii. What is the PV of a cash flow of $100 received in 10 years’ time at discount rate of 5%?

\[
(ii) \quad PV = \frac{$100}{(1+0.05)^{10}} = $61.39
\]

iii. What is the PV of a cash flow of $100 received in 10 years’ time at discount rate of 20%?

\[
(iii) \quad PV = \frac{$100}{(1+0.20)^{10}} = $16.15
\]
The table below shows how the PV of $100 varies with r and t.

<table>
<thead>
<tr>
<th>CF</th>
<th>$100</th>
</tr>
</thead>
<tbody>
<tr>
<td>T/r</td>
<td>1%</td>
</tr>
<tr>
<td>1</td>
<td>$99.01</td>
</tr>
<tr>
<td>2</td>
<td>$98.03</td>
</tr>
<tr>
<td>3</td>
<td>$97.06</td>
</tr>
<tr>
<td>4</td>
<td>$96.10</td>
</tr>
<tr>
<td>5</td>
<td>$95.15</td>
</tr>
<tr>
<td>6</td>
<td>$94.20</td>
</tr>
<tr>
<td>7</td>
<td>$93.27</td>
</tr>
<tr>
<td>8</td>
<td>$92.35</td>
</tr>
<tr>
<td>9</td>
<td>$91.43</td>
</tr>
<tr>
<td>10</td>
<td>$90.53</td>
</tr>
</tbody>
</table>

From the above you can see that at a rate of interest of 50% p.a., the present value of $100 received in 10 years’ time is just $1.73.

### 3.3 Perpetuities and annuities

Perpetuities and annuities are two special cases that we must consider. An annuity is an investment that pays a fixed sum each year for a specified number of years. In contrast, a perpetuity is a financial concept in which a cash flow is theoretically received forever. In Chapter 1 you were introduced to preference shares, a type of share that pays a fixed dividend forever.

The value of a perpetuity can be found by:

$$\text{Present Value of a Perpetuity} = \frac{\text{Cash Flow}}{\text{discount rate}}$$

For example, the value of an asset that promises to pay $100 forever, where the required return is 5% p.a., is:

$$\text{PV of Cash Flow} = \frac{$100}{0.05} = $2000$$

This asset could be, for example, a preference share. If the preference share paid a fixed dividend of $100 per annum, and the market viewed the risk of these cash flows to be 5% p.a.,
then the market value of the preference share would be $2,000. It is evident that as \( r \) increases, the value of the preference share will fall. The example below illustrates how, given the dividend and value of a preference share, we can find the discount rate used by the market to arrive at this price.

A 10\% UK preference share pays out 10\% of a nominal value of 100 pence forever, i.e. it pays out 10p per annum. The price observed in the market is therefore the present value of 10p at a discount rate, which the market views as representing the risk inherent in the future cash flows. It follows then that if we observe the price of the preference share to be 120p, then the market must have used:

\[
P_{MP} = \frac{D}{r} \rightarrow r = \frac{10}{120} = 8.33\% 
\]

It follows that the price of preference shares varies with the market's view of the riskiness of the future cash flows. For example, following the Deepwater Horizon oil rig disaster in April 2010, the price of 8\% BP preference shares fell from around 160 pence to 125 pence. The reason for the drop was that the market felt that the riskiness of future cash flows had risen from 5\% (= 8/160) to 6.4\% (= 8/125).

The value of an annuity can be found using:

\[
\text{PV of annuity} = C \times \left[ \frac{1}{r} - \frac{1}{r(1+r)^t} \right]
\]

The term in brackets is referred to as the annuity factor.

For example, if an investment pays $100 each year for 20 years and the discount rate is assumed to be 5\% p.a., then the value of this investment is:

\[
\text{PV of annuity} = C \times \left[ \frac{1}{r} - \frac{1}{r(1+r)^t} \right] = $100 \times \left[ \frac{1}{0.05} - \frac{1}{0.05(1 + 0.05)^{20}} \right] = $1,246.22
\]
The working out is detailed below:

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>$100</td>
</tr>
<tr>
<td>r</td>
<td>5%</td>
</tr>
<tr>
<td>t</td>
<td>20</td>
</tr>
<tr>
<td>1/r</td>
<td>20</td>
</tr>
<tr>
<td>r(1+r)^t</td>
<td>0.1327</td>
</tr>
<tr>
<td>1/(r(1+r)^t)</td>
<td>7.5378</td>
</tr>
<tr>
<td>Annuity factor</td>
<td>12.4622</td>
</tr>
<tr>
<td>PV</td>
<td>$1,246.22</td>
</tr>
</tbody>
</table>

Alternatively, in Excel, =-pv(0.05,20,100) would have also generated the answer of $1,246.22.

Activity 3.1

Visit the website of the London Stock Exchange\(^1\) and search for “BP.A”. This is a British Petroleum 8% preference share. Display the chart over the last five years and determine the lowest share price and the highest share price. What discount rate was the market using to arrive at these values?

At the time of writing (October 2017), the annuity rate quoted for a single 55-year-old living in London was £4,216. What this figure represents is how much annual income could be purchased with £100,000. Recall that to find the present value of an annuity we need to know the values for \(r\) and \(t\), where \(t\) in this context will represent life expectancy. The Office of National Statistics maintains a database of UK life expectancy according to age and gender which, based on data for the years 2014–16, is presently 24.82 years (for a male aged 55).

If interest rates were 5%, then the PV of £1 received for the next 24.82 years would be:

\[
PV\text{ of annuity} = C \times \left[ \frac{1}{r} - \frac{1}{r(1+r)^t} \right] = £1 \times \left[ \frac{1}{0.05} - \frac{1}{0.05(1 + 0.05)^{26.82}} \right] = £14.5958
\]

\(^1\) http://www.londonstockexchange.com/
Hence if £100,000 was available to buy an annuity, this would be quoted as:

\[ \frac{£100,000}{£14.5958} = £6,851.29 \]

In order to find the discount rate used to arrive at an answer of £4,216 we would need to discount cash flows at a lower r:

<table>
<thead>
<tr>
<th>C</th>
<th>£1.00</th>
</tr>
</thead>
<tbody>
<tr>
<td>r</td>
<td>0.905%</td>
</tr>
<tr>
<td>t</td>
<td>26.82</td>
</tr>
<tr>
<td>1/r</td>
<td>110.5219135</td>
</tr>
<tr>
<td>r(1+r)^t</td>
<td>0.0115</td>
</tr>
<tr>
<td>1/(r(1+r)^t)</td>
<td>86.8027</td>
</tr>
</tbody>
</table>

**Annuity factor** 23.7192

**PV** £23.7192

<table>
<thead>
<tr>
<th>Investment</th>
<th>£100,000.00</th>
</tr>
</thead>
<tbody>
<tr>
<td>Annuity rate</td>
<td>£4,215.99</td>
</tr>
</tbody>
</table>

### 3.4 Dividend discount model

For security analysts and portfolio managers it could be very useful to compute the “equilibrium” price of a security. This value would be especially important if analysts believed that any deviations of the actual price from the theoretical price would only be temporary, so that they could buy temporarily cheap securities and sell temporarily expensive ones. Clearly such strategies are relevant if prices have a tendency to converge towards their fundamental values.

The dividend discount model is based on the premise that the market value of ordinary shares represents the sum of expected future dividends, to infinity, discounted to time zero.

Consider a shareholder who intends to hold a share for one year. A single dividend will be paid at the end of the holding period, \( d_1 \), and the share will be sold at a price \( p_1 \) in one year. To derive the value of a share at time 0 to this investor, the future cash flows \( d_1 \) and \( p_1 \) need to be discounted at a rate which includes an allowance for the risk of the share, \( k \):

\[
p_0 = \frac{d_1}{1 + k} + \frac{p_1}{1 + k}
\]
Consider a second investor who expects to hold the share for a further year and sell at time 2 for \( p_2 \); the price, \( p_1 \), will be:

\[
p_1 = \frac{d_2}{1 + k} + \frac{p_2}{1 + k}
\]

Substituting into the equation for \( p_0 \) we get:

\[
p_0 = \frac{d_1}{1 + k} + \frac{d_2}{(1 + k)^2} + \frac{p_2}{(1 + k)^2}
\]

If a series of one-year investors bought this share, and we in turn solved for \( p_2, p_3 \) etc., we would find:

\[
p_0 = \frac{d_1}{1 + k} + \frac{d_2}{(1 + k)^2} + \ldots \frac{d_\infty}{(1 + k)^\infty}
\]

The terminal stock price can effectively be ignored as its present value is zero. However, its value feeds into \( p_{\infty-1} \) etc. and so by iteration it enters into \( p_0 \).

The above model is called the dividend discount model (DDM) and can be written, using sigma notation, as:

\[
P_t = \sum_{t=1}^{\infty} \frac{d_{t+1}}{(1 + k)^t}
\]

Consider stock A that has just paid a dividend of $10 and is expected to pay this dividend forever. If the cost of capital, \( k \), is 5%, what is the value of stock A?

We could use Excel and find the PV and the future dividends and sum them together, as follows:

<table>
<thead>
<tr>
<th>Year</th>
<th>Div</th>
<th>PV(Div)</th>
<th>Sum (years 1 – 500) = $200</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$10.00</td>
<td>$9.52</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>$10.00</td>
<td>$9.07</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>$10.00</td>
<td>$8.64</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>$10.00</td>
<td>$8.23</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>$10.00</td>
<td>$7.84</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>$10.00</td>
<td>$7.46</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>$10.00</td>
<td>$7.11</td>
<td></td>
</tr>
</tbody>
</table>
Or alternatively we could treat this as a perpetuity and find the price directly using $10/0.05 = $200.

The spreadsheet for this exercise can be found here. Please ensure you click on Section 3 and the 3.4a tab at the bottom of the spreadsheet.

Note, at a discount rate of 5%, it takes 156 years in order for the PV to drop below 1 cent.

Consider stock B that will not pay a dividend for the next five years, but in year six will pay a dividend of $10 and is expected to pay this dividend forever. If the cost of capital, k, is 5%, what is the value of stock B? Again, we can map out the future dividends in Excel, find the PVs and sum them together.

The spreadsheet for this exercise can be found here. Please ensure you click on Section 3 and the 3.4b tab at the bottom of the spreadsheet.
Clearly the answer is lower than that of Stock A. We could have found the answer by taking the value of Stock A and subtracting the sum of the PVs from years 1 to 5 (in fact this is an annuity, for which there is a formula). Or we can think laterally ...

Figuratively speaking, standing at the end of year 5 we know that Stock B will pay a dividend of $10 and pay this dividend forever. In “year five money” the value of Stock B is:

\[ P_5 = \frac{10}{0.05} = 200 \]

In order to get the value in “year zero money” we need to discount at 5% over five years:

\[ P_0 = \frac{200}{1.05^5} = 156.71 \]

3.5 The Gordon growth model

According to the dividend discount model, the equilibrium price of a share is equal to the sum of future dividends, discounted at an appropriate rate of interest to time 0:

\[ P_t = \sum_{t=1}^{\infty} \frac{d_{t+1}}{(1 + k)^t} \]

This model works fine for preference shares, where dividends are known, but for ordinary shares it is less effective. Instead we have to view the equilibrium share price as the sum of the present value of expected future dividends.

\[ P_t = \sum_{t=1}^{\infty} E_{t=0} \left[ \frac{d_t}{(1 + k)^t} \right] \]

That is, the equilibrium share price is the sum of the present value of expected future dividends, with the expectation formed at time 0. In order to overcome the issue of forming expectations, Myron J. Gordon (1962)² simplified the problem by assuming that dividends grow at some rate \( g \), each year.

To motivate the model let us use the following notation:

\[ d_t = \text{the last dividend paid by the company} \]

\[ d_{t+1} = d_t \times (1 + g) \]

It follows that the expectation made at time \( t \) (i.e. year 0) for the dividend received in period 2 is:

\[ E_t(d_{t+2}) = d_{t+1} \times (1+g) \]

And the expectation made at time \( t \) (i.e. year 0) for the dividend received in period 3 is:

\[ E_t(d_{t+3}) = E_t(d_{t+2}) \times (1+g) = d_{t+1} \times (1+g)^2 \]

Hence the expectation made at time \( t \) (i.e. year 0) for the dividend received in period \( i \) is:

\[ E_t(d_{t+i}) = d_{t+1} \times (1+g)^{i-1} \]

Substituting this result into the generic dividend discount model:

\[
P_t = \sum_{t=1}^{\infty} E_t = \frac{d_t}{(1+k)^t} = \sum_{t=1}^{\infty} \frac{d_{t+1} (1+g)^{t-1}}{(1+k)^t} \]

Consider stock C that has just paid a dividend of $10 and is expected to grow dividends at 2% forever. If the cost of capital, \( k \), is 5% what is the value of stock C? Again, we can map out the future dividends in Excel, find the PVs and sum them together.

<table>
<thead>
<tr>
<th>Year</th>
<th>Div</th>
<th>PV(Div)</th>
<th>Sum (years 1 – 500)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$10.20</td>
<td>$9.71</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>$10.40</td>
<td>$9.44</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>$10.61</td>
<td>$9.17</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>$10.82</td>
<td>$8.91</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>$11.04</td>
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<td></td>
</tr>
<tr>
<td>6</td>
<td>$11.26</td>
<td>$8.40</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>$11.49</td>
<td>$8.16</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>$11.72</td>
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<td></td>
</tr>
<tr>
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<td>$11.95</td>
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<td></td>
</tr>
<tr>
<td>10</td>
<td>$12.19</td>
<td>$7.48</td>
<td></td>
</tr>
</tbody>
</table>

Clearly the answer is larger than stock A ($200) as dividends are growing by 2% each year.

The spreadsheet for this exercise can be found [here](#). Please ensure you click on Section 3 and the 3.5 tab at the bottom of the spreadsheet.
It is not particularly practical to implement this approach without a spreadsheet. But with some algebraic manipulation we can arrive at a simple equation.

\[ P_t = \sum_{t=1}^{\infty} \frac{d_{t+1}(1 + g)^{t-1}}{(1 + k)^t} \]

If we can get the summation sign from zero to infinity rather than from 1 to infinity, we can treat the result as a geometric series. Note, if we start summing from 1 we must increase the index within the summation by 1 to give:

\[ P_t = \sum_{t=0}^{\infty} \frac{d_{t+1}(1 + g)^t}{(1 + k)^{t+1}} \]

Note that we can remove the \(d_{t+1}\), from the summation as it does not have an “i” index and we can simplify further:

\[ P_t = d_{t+1} \sum_{t=0}^{\infty} \frac{(1 + g)^t}{(1 + k)^{t+1}} = d_{t+1} \sum_{t=0}^{\infty} \frac{(1 + g)^t}{(1 + k)^t(1 + k)} \]

\[ P_t = \frac{d_{t+1}}{(1 + k)} \sum_{t=0}^{\infty} \frac{(1 + g)^t}{1 + k} = \frac{d_{t+1}}{(1 + k)} \sum_{t=0}^{\infty} (1 + \frac{g}{1 + k})^t \]

The last expression on the right-hand side is a geometric expression. If \( g < k \) so that the entire bracketed term < 1, we can use the following rule:

\[ \sum_{i=0}^{\infty} a^i = \frac{1}{1 - a} \text{ if } a < 1 \]

Then:

\[ P_t = \frac{d_{t+1}}{(1 + k)} \times \frac{1}{1 - \left(\frac{1+g}{1+k}\right)} = \frac{d_{t+1}}{k - g} \]

i.e. the equilibrium share price is the next period dividend divided by \( k \) minus \( g \). For Stock C:

\[ P_t = \$10 \times \frac{1.02}{0.05 - 0.03} = \frac{10.2}{0.02} = \$340. \]

**Example**

Stock A is expected to pay a dividend of £10 forever.

Stock B is expected to pay a dividend of £8 next year, with dividend growth expected to be 3% per annum thereafter.
Stock C just paid a dividend of £6 with dividend growth expected to be 4% p.a. thereafter.
If the required return on similar equities is 10%, calculate the value of each stock.

Solution

A: £10/0.1 = £100
B: £8/(0.1 – 0.03) = £8/0.07 = £114.29
C: (£6 x 1.04)/(0.1 – 0.04) = £6.24/0.06 = £104

3.6 Two-period dividend growth model

What if we do not anticipate a constant rate of growth of dividends? In this case we would need to forecast all future dividend payments, which is of course impossible. One solution is to assume that dividends grow at a constant rate over a certain period and then grow at a different rate over a second period etc. Clearly the simplest case to solve is the two-period growth model in which dividends grow at $g_1$ in periods 1 to $N$ and then at $g_2$ in periods $N + 1$ to infinity. This is best illustrated by way of an example.

Consider stock D that has just paid a dividend of $10 and is expected to maintain this stable dividend for the next five years, and then grow dividends at 2% forever. If the cost of capital, $k$, is 5%, what is the value of stock D? Again, we can map out the future dividends in Excel, find the PVs and sum them together.

<table>
<thead>
<tr>
<th>Year</th>
<th>Div</th>
<th>PV(Div)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$10.00</td>
<td>$9.52</td>
</tr>
<tr>
<td>2</td>
<td>$10.00</td>
<td>$9.07</td>
</tr>
<tr>
<td>3</td>
<td>$10.00</td>
<td>$8.64</td>
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<td>4</td>
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<td>$8.23</td>
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<td>$10.00</td>
<td>$7.84</td>
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<tr>
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<tr>
<td>8</td>
<td>$10.61</td>
<td>$7.18</td>
</tr>
</tbody>
</table>

Sum (years 1 – 500) = $309.69

Sum = $43.29
Clearly the answer is larger than stock A ($200), as dividends are growing by 2% each year, but less than stock C, as growth was delayed until the sixth year.

The spreadsheet for this exercise can be found here. Please ensure you click on Section 3 and the 3.6a tab at the bottom of the spreadsheet.

Once again it is not particularly practical to implement this approach without a spreadsheet. However, with some lateral thinking it is possible to solve the problem easily.

In “year five money” the value of Stock D’s cash flows in year 6 and beyond is:

\[ P_5 = \frac{\$10 \times 1.02}{0.05 - 0.02} = \$340 \]

In order to get the value in “year zero money” we need to discount at 5% over five years:

\[ P_0 = \frac{\$340}{1.05^5} = \$266.40 \]

But then, of course, we have to consider the PV of the dividends received in years 1 to 5:

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<td>4</td>
<td>$10.00</td>
<td>$8.23</td>
</tr>
<tr>
<td>5</td>
<td>$10.00</td>
<td>$7.84</td>
</tr>
</tbody>
</table>

\[ \text{Sum} = \$43.29 \]

giving a total of $266.40 + $43.29 = $309.69.

More formally we can determine the dividend in each period as:

\[ d_{t+1} = d_t(1 + g_1) \]
\[ E_t(d_{t+2}) = d_{t+1}(1 + g_1) \]
\[ E_t(d_{t+3}) = E_t(d_{t+2})(1 + g_1) = d_{t+1}(1 + g_1)^2 \]
\[ E_t(d_{t+N}) = E_t(d_{t+N-1})(1 + g_1) = d_{t+1}(1 + g_1)^{N-1} \]
\[ \ldots \]
\[ E_t(d_{t+N+1}) = E_t(d_{t+N})(1 + g_2) = d_{t+1}(1 + g_1)^{N-1}(1 + g_2) \]
\[ E_t(d_{t+N+2}) = E_t(d_{t+N+1})(1 + g_2) = d_{t+1}(1 + g_1)^{N-1}(1 + g_2)^2 \]
\[ \ldots \]
\[ E_t(d_{t+N+i}) = d_{t+1}(1 + g_1)^{N-1}(1 + g_2)^i \]


In the example above, \( d_t = $10, g_1 = 0\% \) (therefore \( d_{t+1} = $10 \)), \( g_2 = 2\% \) and \( N = 5 \). Therefore the dividend, in say period 8, can be found using:

\[
E_t(D_{t+N+i}) = d_{t+1}(1 + g_1)^{N-1}(1 + g_2)^i = E_t(d_{t+5+3}) = 10 \times (1 + 0)^4 \times (1 + 0.02)^3 = $10.61
\]

Consider stock E that has just paid a dividend of $10 and is expected to grow this dividend at a rate of 10% p.a. for the next five years, and then grow dividends at 2% forever. If the cost of capital, \( k \), is 5%, what is the value of stock E?

Again, we can map out the future dividends in Excel, find the PVs and sum them together.

\[
\begin{array}{c|c|c|c}
\text{Year} & \text{Div} & \text{PV(Div)} & \text{Sum (years 1 – 500)} = \$486.65 \\
1 & $11.00 & $10.48 & \\
2 & $12.10 & $10.98 & \\
3 & $13.31 & $11.50 & \\
4 & $14.64 & $12.05 & \\
5 & $16.11 & $12.62 & \\
6 & $16.43 & $12.26 & \\
7 & $16.76 & $11.91 & \\
8 & $17.09 & $11.57 & \\
9 & $17.43 & $11.24 & \\
10 & $17.78 & $10.92 & \\
\end{array}
\]

Here, we map out the dividends in the “finite” period of years 1–5 as above and find the PV to be $57.61. Next, we move to year 5 where the dividend is expected to be $16.11 after five years of growth at 10%. The dividend in period 6 is then:

\[
$16.11 \times 1.02 = $16.43
\]

Using the single-period growth model, the value of the cash flows in year 6 and beyond (in year 5 money) is then:

\[
$16.43/(0.05 – 0.02) = $547.67
\]

In order to compare this to the $57.61 figure obtained earlier, we need to discount these cash flows back to year 0:
$547.67/(1 + 0.05)^5 = $ 429.11

The value of stock E is then $57.61 + $429.11 = $486.72. Note that the slight discrepancy in the answer above is due to rounding.

The spreadsheet for this exercise can be found here. Please ensure you click on Section 3 and the 3.6b tab at the bottom of the spreadsheet.

3.7 Example with earnings growth

Vornado is a company that has patent rights for a new mobile phone technology that is expected to enable it to generate growth in earnings of 20% for the next three years. After that (from the start of year 4) the company expects to see earnings growth drop to a constant rate of 5%. Assuming that the company pays out 60% of earnings as dividends and that the last dividend payment made by the company was $2.20, calculate an estimate of the current price of Vornado. Assume that the required return on equity is 8%.

This is rather a difficult exercise as it has a number of complexities relative to previous examples. Was the reference to the company paying out 60% of earnings as dividends and growing earnings by 20% a “red herring”? Let’s assume some earnings and share values.

Year 0
Earnings = $3,666,666.67
Number of shares = 1,000,000

What is the total dividend and the dividend per share?

Dividend = 60% x $3,666,666.67 = $2,200,000
Dividend per share = $2.20

Note we arrive at the same answer if the number of shares is 500,000 and the earnings are $1,833,333.33.

If the firm grows earnings at 20% per annum, then the new earnings level, total dividend and dividend per share are as follows:

Year 1
Earnings = $4,400,000.00,
Dividends = $2,640,000.00
Dividend per share = $2.64

Dividends have increased from $2.20 to $2.64, which is an increase of \([2.64 – 2.20]/2.20 = 20\%\]
Hence, if a firm is to pay a constant proportion of earnings, then earnings growth and dividend growth are the same. For example:

\[
DPS_0 = \frac{c \times Earnings}{n}
\]

\[
DPS_1 = \frac{c \times Earnings \times (1 + g)}{n}
\]

\[
\frac{DPS_1 - DPS_0}{DPS_0} = \frac{c \times Earnings \times (1 + g) - c \times Earnings}{c \times Earnings} = g
\]

The case can now be expressed as:

Vornado is a company that has patent rights for a new mobile phone technology that is expected to enable it to generate growth in DIVIDENDS of 20% for the next three years. After that (from the start of year 4) the company expects to see DIVIDENDS’ growth drop to a constant rate of 5%. The last dividend payment made by the company was $2.20; calculate an estimate of the current price of Vornado. Assume the required return on equity is 8%.

Now we need to map out the future stream of dividends:

<table>
<thead>
<tr>
<th>Year</th>
<th>Dividend</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$2.64</td>
</tr>
<tr>
<td>2</td>
<td>$2.64 + 20% = $3.17</td>
</tr>
<tr>
<td>3</td>
<td>$3.17 + 20% = $3.80</td>
</tr>
<tr>
<td>4</td>
<td>$3.80 + 5% = $3.99</td>
</tr>
</tbody>
</table>

In year 4, dividend growth drops to 5% and continues to grow at 5% into infinity.

Finding the sum over the first three periods is trivial:

\[
\frac{2.64}{1.08} + \frac{3.17}{1.08^2} + \frac{3.80}{1.08^3} = 8.18.
\]

In order to find the value of the dividends beyond period 3 we need to use the Gordon growth model:

\[
\frac{3.99}{0.08 - 0.05} = 133
\]
However, that is in “year 3 money”, and so to find the value now we must find its present value (PV):

$$\frac{133.00}{1.08^3} = 105.58$$

Adding this to the PV of the dividends from years 1 to 3 we get a current share price of:

$$8.18 + 105.58 = 113.76$$

### 3.8 Real-life dividend policy

The fact that corporations and shareholders expend a considerable amount of energy on analysing a corporation’s dividend policy suggests that dividend payments have a significant effect on the share price or value of the company. It was therefore surprising that in 1961 Modigliani and Miller published an article, “Dividend Policy, Growth, and the Valuation of Shares”, which demonstrated that in certain circumstances a corporation’s decision regarding its dividend payments would have no effect on the value of its shares. Prior to this article, it was widely accepted that the more dividends a firm paid, the higher would be its value, as we see in the dividend discount model.

There have been a number of arguments proposed to explain why dividends might be relevant. Below we outline three of these.

1. **Tax and clientele effect.** In many countries, the investor receiving dividends is taxed at a higher rate than the investor who obtains a capital gain by selling shares. Hence investors might prefer to receive their share of the firm’s profit in the form of a capital gain rather than in the form of a dividend. Moreover, even if realised capital gains and dividends incur the same tax rate, investors may prefer a firm not to pay a dividend because, although that would lead to a higher share price, the investor can postpone tax by delaying the sale of the share. This is because tax is charged on capital gains only when the share is sold and the capital gain realised, so the investor is able to postpone tax liability and benefit from retaining the use of the tax funds. In contrast, tax on dividend income cannot be postponed. Therefore dividend policy is not irrelevant, and firms have an incentive not to pay dividends so as to help their shareholders avoid taxation. In reality, however, investors are taxed differently and the fact that investors are subject to different taxes can lead to their forming clienteles based on their tax bracket. Investors in high tax brackets invest in stock with low dividend payouts, while those in low tax brackets invest in high-dividend-paying

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stocks. The gravitation of investors towards companies with dividend policies more suited to their own individual tax situations is often referred to as the “clientele effect”. Tax clienteles are different types of shareholders, each preferring a specific kind of dividend policy due to differences in their tax brackets. The clientele effect suggests that with diverse investors, the value of the firm is independent of its payout policy. If a firm increases its dividend payout, it may lose some shareholders, but then other investors, who prefer this new dividend policy, will replace them.

2. **Cliente effect again.** It can be further argued that an investor’s attraction to a company will depend on their current financial situation. For example, if an investor has more than enough money from their paid employment and would probably only reinvest dividend income in the stock market, then they will be attracted to stocks with a low dividend payout rate/high reinvestment rate. In comparison, an investor who relies on dividend income to live on would be attracted to stocks with a high dividend payout rate. Pension funds, which need regular income, would also be attracted to such stocks. Interestingly, the dividend discount models outlined above rely on the fact that companies pay a dividend. Apple, for example, did not pay a dividend from December 1995 to August 2012. In contrast, British American Tobacco grows its annual dividend year on year.

<table>
<thead>
<tr>
<th>Date</th>
<th>BAT annual dividend</th>
</tr>
</thead>
<tbody>
<tr>
<td>1999</td>
<td>20.3</td>
</tr>
<tr>
<td>2000</td>
<td>26.9</td>
</tr>
<tr>
<td>2001</td>
<td>29.7</td>
</tr>
<tr>
<td>2002</td>
<td>33</td>
</tr>
<tr>
<td>2003</td>
<td>36.3</td>
</tr>
<tr>
<td>2004</td>
<td>39.7</td>
</tr>
<tr>
<td>2005</td>
<td>43.2</td>
</tr>
<tr>
<td>2006</td>
<td>48.7</td>
</tr>
<tr>
<td>2007</td>
<td>58.8</td>
</tr>
<tr>
<td>2008</td>
<td>69.7</td>
</tr>
<tr>
<td>2009</td>
<td>99.5</td>
</tr>
<tr>
<td>2010</td>
<td>114.2</td>
</tr>
<tr>
<td>2011</td>
<td>126.5</td>
</tr>
<tr>
<td>2012</td>
<td>130.6</td>
</tr>
<tr>
<td>2013</td>
<td>142.4</td>
</tr>
<tr>
<td>2014</td>
<td>148.1</td>
</tr>
</tbody>
</table>
In percentage terms British American Tobacco has grown the dividend by:

\[ g = \sqrt[18]{\frac{174.6}{20.3}} - 1 = 12.70\% \text{ per annum.} \]

3. **Asymmetric information.** The Modigliani–Miller “dividend irrelevance” result was derived within a model of perfectly competitive markets, including the assumption that there is perfect information – that financial investors (shareholders) and managers have the same, freely available information on which to base their valuation of the corporation. This does not mean that variables are known with complete certainty, but the point is that all parties have the same knowledge of those uncertain variables. In reality, however, shareholders and managers may have different degrees of knowledge about the corporation. It follows that when shareholders have imperfect information, managers can choose to use dividends as a signal to convey information about the company. Therefore, if information is asymmetric, a company could try and convey confidence about the future to financial markets, which it could do by increasing the dividend beyond the increase expected by the market. In contrast, if the firm chose to reduce the dividend it would convey a negative signal about its future prospects. Consequently, dividends can affect the value of the corporation by influencing shareholders’ perception and valuation of its prospects and risks; consequently, the choice of dividend policy does matter.

### Activity 3.2

Search online for the following terms: “Investor relations Amazon”, “Investor relations Alphabet” and “Investor relations Imperial Brands” and obtain the most recent annual reports. Search the annual report (CTRL + F) for the word “dividend”. What is the dividend policy for each of these companies?