

The Effect of Risk Aversion on Manufacturer Advertising in a Two-Stage Supply Chain

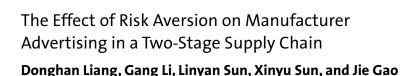
Donghan Liang, Gang Li, Linyan Sun, Xinyu Sun, Jie Gao

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Abstract

We consider a supply chain system with a risk-neutral manufacturer as the leader and a risk-averse retailer as the follower with uncertain demand. At the beginning of the game, the manufacturer makes efforts on advertising and then the retailer decides its order quantity before demand realization. The retailer's risk aversion is modeled by the Value-at-Risk (VaR) approach with the downside risk constraint. The analysis of equilibrium strategies indicates some characteristics of the game are different from those under risk-neutral assumptions. We find that the manufacturer can effectively prevent the risk-averse retailer from downsizing the order quantity through advertising. In order to explain the difference, we investigate the impacts of risk aversion on the manufacturer's advertising decision and the retailer's ordering decision. We find that the retailer with moderate degree of risk aversion orders a larger volume and receives greater advertising support from the manufacturer. Moreover, the feasible combinations of target profit and downside risk for moderate risk aversion are discussed to derive the relationship of the two parameters. In addition, we make a simple analysis of the situation with two independent retailers who have heterogeneous degrees of risk aversion.

Keywords

Risk aversion; Value-at-Risk (VaR); Stackelberg game; Newsvendor

Donghan Liang E-mail: ldhsweet@yahoo.com.cn

Gang Li E-mail: lee_rich@163.com

Linyan Sun E-mail: lysun@mail.xjtu.edu.cn

Xinyu Sun

E-mail: Igtxysun@inet.polyu.edu.hk Department of Logistic and Maritimes Studies The Hong Kong Polytechnic University, Hung Hom, Hong Kong **Jie Gao** E-mail: gaoj@mail.xjtu.edu.cn

Liang, Li, L. Sun, and Gao The Management School of Xi'an Jiaotong University The State Key Lab for Manufacturing Systems Engineering The Key Lab of the Ministry of Education for Process Control & Efficiency Engineering, Xi'an, 710049, China

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Introduction

The power of promotion has been identified in abundant literature on marketing as well as supply chain management. Promotional activities are implemented by suppliers as well as retailers in various manners, ranging from media advertising, events sponsorship, catalogues distribution, to salespeople's effort. In model analysis, advertising is widely discussed as a typical promotion activity and classified into "brand advertising" and "local advertising." Usually, brand advertising is implemented by brand owners to make their products less substitutable and to earn more profit (Shaffer and Zettelmeyer 2004). More specifically, manufacturers expect to grasp potential demand and to develop brand knowledge and customer preference through brand advertising (Huang and Li 2001). In the fiscal year of 2009, Apple's advertising budget reached \$501 million as disclosed in the Form 10-K document submitted to the Securities and Exchange Commission of America. According to Fortune magazine, Dell spent \$811 million on advertising in the fiscal year 2009, while Microsoft's spending was up to \$14 billion. These industry giants make many efforts on advertising for both long-term and short-term returns, revealing the leading position of advertising among numerous marketing tools.

Most marketing studies to date focus on the performance of advertising strategy or customers' responses to retailers' sales effort. Another related topic is about vertical co-op advertising, the scheme in which supply chain members share the cost of local advertising. Based on the decision-making process or the game sequence, these articles (Huang and Li 2001; Jørgensen, Taboubi, and Zaccour 2003; Karray and Zaccour 2006) target the coordination of the entire supply chain performance with Pareto optimality. Generally speaking, researchers are more interested in improving advertising efficiency under classical assumptions that perfect rationality and risk neutrality are the building blocks for model construction. The emergence of behavioral economics has shed some light on this subject and proposes hypotheses that are more aligned to the decision behaviors in the real world. Many experiments and empirical studies have proved the existence of biases in decision-making, thus challenging the conventional wisdom about the supply chain optimizing solution. This article studies the mutual effects of the retailer's risk aversion and the manufacturer's advertising in a supply chain. We find that the equilibrium of the game is different from those of the classical model conducted under the assumption that both the manufacturer and the retailer are risk neutral. These findings give new insights into the role of advertising in supply chains.

Proved to be influential in decision-making, the risk preference of supply chain members plays an important role in supply chain management research (e.g., Agrawal and Seshadri 2000; Chen et al. 2009). Recent studies on retailers' ordering and pricing policies pay much attention to retailers' risk preferences, especially risk aversion, to acquire meaningful insights for improving supply chain efficiency (Jammernegg and Kischka 2009; Tapiero and Kogan 2009). Early measures used to describe a degree of risk aversion include utility maximization and mean-variance analysis, both of which are still in use and illustrative for a number of scenarios. Lately, the introduction of financial measures such as Value-at-Risk (VaR) and Conditional Value-at-Risk (CVaR) has directly bridged risk aversion with target profit and made risk aversion more measurable. These metrics have greatly enriched the research on risk aversion and made analysis more applicable to real world operations. So far supply chain management literature using VaR and CVaR approaches mainly focuses on the inventory issues with little consideration of the impact of marketing tools. As an exploratory study, this article takes the effect of advertising into account and investigates the relationship between promotional activities and risk aversion.

This research follows the work of Gan's (2005) on supply chain coordination with a risk-averse retailer and a risk-neutral manufacturer, where the VaR theory is employed in a constrained condition to measure the degree of the retailer's risk aversion. We follow most of the assumptions in Gan's work and investigate the equilibrium strategies from the manufacturer's perspective. The aim of this study is to understand the impact of brand advertising on retailers' decision-making. We construct a Stackelberg game in which the manufacturer acts as the leader and the retailer acts as the follower. The results suggest that the risk-neutral manufacturer increases promotional effort in a certain range determined by the degree of the retailer's risk aversion, and that a risk-averse retailer does not necessarily order less than a risk-neutral one.

The article is organized as follows. We first review related literature. Then we propose our assumptions, propose a model with a downside risk constraint, and report the equilibrium strategies. Next, we extend the model to the situation with two independent retailers that have different target profits and downside risks. Finally, we conclude and discuss managerial implications.

Literature Review

Advertising has been widely investigated in both marketing and supply chain management research. In the literature there are many papers proposing various advertising strategies from different points of view. One branch of marketing studies focuses on advertising attributes such as information, contents, and exhibit modes (e.g., Bloch and Manceau 1999; Dukes and Gal-Or 2003). They study the performance of advertising in relation to customer choice and market demand, trying to improve advertising quality to cater to public taste. The other branch considers advertising as a pure marketing strategy, studying the efficiency of advertising in fierce market competition (e.g., Narasimhan, Neslin, and Sen 1996; Banerjee and Bandyopadhyay 2003). Usually the marketing perspective on advertising is customer-oriented, mainly concerning the performance and influencing factors of advertising as a market tool. The seller's risk preference and other industrial behavioral features are rarely included in such analysis.

Advertising is classified into brand advertising and local advertising in most supply chain management literature. Brand advertising entails the costly differentiation efforts paid by manufacturers to foster customer loyalty (Baye and Morgan 2009), while local advertising consists of promotional activities implemented around retailers' outlets. Because the supply chain management perspective emphasizes the interaction between upstream and downstream players, local advertising has been discussed much more frequently than brand advertising. In the line of supply chain coordination research, local advertising is considered an important factor controlled by the retailer to promote the market demand. As a result, the manufacturers need to propose well-designed contracts to encourage the retailers to invest in sales promotion. Cachon (2002) thoroughly reviewed supply chain coordination studies on newsvendor with a demand which is dependent on retailers' promotional efforts. He proposed the necessary conditions under which the supplier would share the retailer's promotion expense to achieve supply chain coordination. The condition indicates that the promotional cost should be observable to the supplier, verifiable to the third party, and directly beneficial to the supplier. Otherwise, the costsharing contract cannot be implemented.

Constrained by this rule, most studies either give particular demonstrations about the definition of the promotional effort in their model or directly choose advertising as the promotional parameters for their observable and verifiable cost structure. However, in the supply chain coordination literature, the retailer's promotional activities have been widely investigated, while the manufacturer's brand advertising has been neglected. Netessine and Rudi (2000) presented a coordinating contract that integrated the advertising cost sharing and revenue sharing contract to achieve coordination. Wang and Gerchak (2001) assumed the demand for a certain product is influenced by its display level, which is arranged by the retailer. They indicated that the manufacturer should compensate the retailer with an extra holding cost to coordinate the channel. Taylor (2002) proposed a supply chain coordination contract in which the retailer receives an extra rebate from the manufacturer if the sales exceed a target quantity. The paper proved that the retailer would choose Pareto optimal promotion effort, given a proper target quantity and rebate rate. Krishnan, Kapuscinski, and Butz (2004) extended Taylor's research to a dynamic game process. Assuming that the retailer chooses inventories ex ante and promotional effort ex post, they investigated various coordination mechanisms for different scenarios. In summary, this line of research concerns the appropriate compensation mechanism for the retailer who bears the cost of promotion that is beneficial to the supplier as well. The contracts designed for this purpose are based on the reallocation of the cost and profit to ensure the compensation would reduce the retailer's risk without compromising the effort incentives. Although these studies derived various kinds of contracts, they are based on the same theory that the manufacturer should share the retailer's risk in both inventory holding and advertising investment aspects. However, the fact that these studies discuss risk sharing under the risk-neutral assumption suggests that the research can be improved from the risk-preference point of view.

Besides advertising studies, another stream of literature is pertinent to risk aversion in supply chain management. Many studies take risk aversion into consideration in the area of inventory management. Early research used expected utility function and mean-variance measure to evaluate risk aversion (e.g., Eeckhoudt, Gollier and Schlesinger 1995; Chen et al. 2007; Tapiero and Kogan 2009). Recently, new methods, including VaR and CVaR, were developed with the introduction of financial measures of risk management. VaR measures the player's maximum profit at a specified confidence level (Jorion 2006). It directly combines the profit with risk aversion. Due to the complex computational characteristics, there are limited research studies using the VaR measure. Gan, Sethi, and Yan (2005) solved a newsvendor model with a VaR constraint for the retailer's optimal order quantities. They also designed a contract for the risk-neutral manufacturer to cooperate with a risk-averse retailer. Ozler, Tan, and Karaesmen (2009) extended

the model constructed in Gan, Sethi, and Yan's work to multiproduct scenarios. They derived the exact distribution function for the two-product newsvendor problem and developed an approximation method for the N-product case. These studies also pay attention to risk sharing in dealing with risk aversion; however, few study the impact of marketing power on risks. Huang and Li (2001) developed three deterministic models to explain a cost-sharing scheme between a manufacturer and a retailer. For the cooperative model, they employed a Nash bargaining game and took supply chain members' risk preference into account. Utilizing the Pratt-Arrow risk aversion function, they found that the manufacturer shares a smaller part of the local advertising cost if the retailer has a higher degree of risk aversion. Suo, Wang, and Jin (2005) presented a model that considers retailer's loss aversion. They found that loss aversion would weaken the retailer's incentives for sales effort and the retailer's optimal effort level decreases as loss aversion increases. Yet none of these studies gives a thorough discussion on the interacting effect between the manufacturer's marketing strategy and the retailer's risk preference.

In summary, most of the extant literature held the retailer perspective and emphasized the impact of downstream power on demand. In contrast, the upstream advertising is rarely referred to as a major parameter in the field of supply chain management. Real-world manufacturer advertising has a remarkable effect on all members of the supply chain, especially when retailers' risk aversion is involved. This article develops a two-stage newsvendor model with a risk-averse retailer and a risk-neutral manufacturer. In our model, the manufacturer's advertising is considered to illustrate the significant role that the manufacturer's marketing strategy plays in risk sharing between upstream and downstream collaborations. As the VaR method is far from widely used in this criterion, we will follow the work of Gan, Sethi, and Yan (2005) to analyze the retailer's risk aversion and the players' decision-making process.

The Two Stage Newsvendor Model

Model Description

We now consider a Stackelberg game that consists of a risk-neutral manufacturer M and a risk-averse retailer R. In the game, M performs as the leader and R plays as the follower. Based on the newsvendor model, we suppose the transaction contains a single perishable product with a random market demand X (i.e., the deterministic quantity of X cannot be observed before the selling season). This random market demand has a probability distribution density f(x) and cumulative distribution function F(x), both of which are known to both the manufacturer and the retailer. The timing sequence of the game is as follows. First, the manufacturer promotes its product with an advertising level ρ to enlarge the market demand at an expense $V(\rho)$. $V(\rho)$ increases on ρ with $V(\rho) \ge 0$, $V(\rho) \ge 0$. Advertising extends the original demand X to ρX when the selling season begins. Then, the manufacturer wholesales products to the retailer at a unit cost c and receives w for each unit, and the retailer will sell them to the market at a price p per unit. Finally, the selling season begins and the realized market demand ρX is observed. For the simplicity of our analysis, we assume the goodwill cost and salvage value of the perishable product are zero for both players. We also assume that each player targets at optimizing its expected profit within the constraint and there is no information asymmetry.

There are two critical decision variables in the system: the manufacturer's advertising level ρ and the retailer's order quantity q. The following analysis will focus on these two variables. Our model mostly inherits the traditional newsvendor model with promotion effort involved. The retailer's risk aversion is transformed to a downside risk constraint presented in this part. The concept of downside risk was introduced in Gan, Sethi, and Yan's model (2005). It is a probability that implies the biggest bearable failure rate when the agent cannot achieve his or her target profit. As a result, the retailer would keep the order quantity under a certain level to prevent the downside risk from exceeding the probability. According to the definition, we derive the constraint condition. Suppose the retailer has a target profit α and downside risk β , its risk constraint can be written as:

$$\mathbb{P}(\prod_{r} \le \alpha) \le \beta \tag{1}$$

where $\prod_{r} = p \min(q, \rho X) - wq$ represents the retailer's profit.

The expected profit functions for the manufacturer, the retailer, and the supply chain system are defined as the following:

$$\pi_m = (w - c)q - V(\rho) \tag{2}$$

$$\pi_r = pE\min(q, \rho X) - wq \tag{3}$$

$$\pi_{s} = \pi_{m} + \pi_{r} = pE\min(q, \rho X) - cq - V(\rho)$$
(4)

Then we solve for the distributional equilibrium strategies with the manufacturer as the leader and the retailer as the follower. The result is Stackelberg equilibrium.

Equilibrium Strategies

We begin with solving the retailer's order quantity with the downside risk constraint. Let q^* be the optimal order quantity of the retailer whose maximization problem is described in (P_i) :

$$\max_{q \ge 0} \pi_r = p E \min(q, \rho X) - wq$$

s.t. $P(p \min(q, \rho X) - wq \le \alpha) \le \beta$ (P₁)

The manufacturer's maximization problem is defined as follows:

$$\max_{\rho \ge 1} \pi_m = (w - c)q - V(\rho) \qquad (P_2)$$

Without loss of generality, we split the downside risk constraint into two scenarios: $q \le \rho X$ and $q > \rho X$ (in which the variable ρ is treated as a known constant because it has been decided by the manufacturer at the first stage of the game). For the first scenario, all products are sold out and constraint (1) is equal to the expression below:

$$P((p-w)q \le \alpha) \le \beta \tag{5}$$

Therefore, we get the lower bound of the retailer's optimal order quantity as $q^{\circ} = \frac{\alpha}{p-w}$. The retailer makes a profit of no more than (p-w)q given its order quantity q. As a result, if the order quantity is less than q° , the target profit α can never be achieved. It follows that the retailer has to order at least q° to meet its target profit. When $q^{\circ} < q \le \rho X$, the retailer would gain a profit higher than α and the downside risk is zero; therefore, the constraint binds only if $q > q^{\circ}$ and $q > \rho X$.

For the second scenario $q > \rho X$, we have:

$$P(p\rho X - wq \le \alpha) = P\left(X \le \frac{\alpha + wq}{p\rho}\right) = F\left(\frac{\alpha + wq}{p\rho}\right) \le \beta$$
(6)

Expression (6) relates the demand distribution function F(x) to the downside risk β . With some manipulation on expression (6), we get an upper bound of q as $q \leq \frac{p\rho F^{-1}(\beta) - \alpha}{w}$. As there are two parameters concerning the degree of retailer's risk aversion, we must consider every possible combination of α and β . Therefore, we divide the scope of β into three regions to get specific equilibrium strategies.

Let ρ^* be the optimal advertising level invested by the manufacturer in the first stage and $(\bar{\rho}, \bar{q})$ be the equilibrium strategy for traditional newsvendor (all the players are risk neutral). Let (ρ_h^*, q_h^*) be the equilibrium strategy for a larger β that $\beta \ge F\left(\frac{\alpha + w\overline{q}}{p\overline{\rho}}\right)$ and $\left(\rho_{l}^{*}, q_{l}^{*}\right)$ for the smaller β that $F(q^{\circ}) < \beta < F\left(\frac{\alpha + w\overline{q}}{p\overline{\rho}}\right)$. Theorem 1 describes the equilibrium strategy $\left(\rho^{*}, q^{*}\right)$ with parameters α and β in different regions.

Theorem 1: The equilibrium order quantity and advertising level are as follows:

If
$$o < \alpha \leq \frac{\overline{\rho}}{\overline{\rho} - \left(1 - \frac{w}{p}\right)} \cdot \frac{w}{p} (p - w) F^{-1} \left(\frac{p - w}{p}\right)$$
, then
(1) when $\beta \geq F \left(\frac{\alpha + w\overline{q}}{p\overline{\rho}}\right)$, $q_h^* = \overline{q} = \overline{\rho} F^{-1} \left(\frac{p - w}{p}\right)$, $\rho_h^* = \overline{\rho}$,
 $V'(\rho)|_{\rho = \overline{\rho}} = (w - c) F^{-1} \left(\frac{p - w}{p}\right)$;
(2) when $F(q^\circ) < \beta < F \left(\frac{\alpha + w\overline{q}}{p\overline{\rho}}\right)$, $q_l^* = \frac{p\rho_l^* F^{-1}(\beta) - \alpha}{w}$,
 $V'(\rho)|_{\rho = \rho_l^*} = \frac{(w - c)p}{w} F^{-1}(\beta)$

(3) when $\beta \leq F(q^{\circ})$, there is no equilibrium solution.

If
$$\frac{\overline{\rho}}{\overline{\rho} - \left(1 - \frac{w}{p}\right)} \cdot \frac{w}{p} (p - w) F^{-1} \left(\frac{p - w}{p}\right) < \alpha \leq \overline{\rho} (p - w) F^{-1} \left(\frac{p - w}{p}\right)$$

then the equilibrium strategy is $(\overline{\rho}, \overline{q})$ with any β that satisfies $F(q^{\circ}) < \beta < 1$.

If
$$\alpha > \overline{p}(p - w)F^{-1}\left(\frac{p - w}{p}\right)$$
, there is no available solution.

Proof: All proofs are provided in the appendix.

Note that the retailer's target profit cannot exceed its highest revenue in the risk-neutral setting; otherwise there is no appropriate order quantity that satisfies the downside risk constraint. Theorem 1 also indicates that if the retailer raises the target profit, it has to simultaneously prepare for bearing higher downside risk to allow for available solutions. When the retailer is highly risk-averse with high target profit and low downside risk, it is almost impossible to get equilibrium strategies because the lower-bound constraint $\beta > F(q^\circ)$ is violated. Moreover, with a high degree of risk aversion, the transaction cost (e.g., transportation fee, time cost, and opportunity cost) would be relatively expensive for both parties in the deal. In this situation, there is no equilibrium strategy just like the third proposition in Theorem 1.

Theorem 1 shows all the possible solutions for the game in which we are interested in several regions with closed-form expressions. These equilibrium strategies are derived for the retailer with moderate degree of risk aversion, that is, higher target profit with bigger downside risk or lower target profit with smaller downside risk. In fact, companies usually prefer medium- or low-risk aversion because the limited resource could be fully invested into profit-making. In contrast, a highly risk-averse company would have resources unnecessarily occupied to prepare for the rainy days.

The Impact of Risk Aversion on Advertising Level

An important issue is the change caused by the introduction of manufacturer's advertising effort. Theorem 2 will specifically investigate the advertising variable ρ^* and make comparison of its value when the retailer's target profit and downside risk changes.

Theorem 2: The optimal advertising level changes with the retailer's risk aversion rate are as follows:

When
$$0 < \alpha \leq \frac{\overline{\rho}}{\overline{\rho} - \left(1 - \frac{w}{p}\right)} \cdot \frac{w}{p} \left(p - w\right) F^{-1} \left(\frac{p - w}{p}\right)$$
 and $\beta > F(q^{\circ})$,

the comparison of ρ_l^* and ρ_h^* ($\rho_h^* = \overline{\rho}$) are as follows:

$$\operatorname{If} \frac{w}{p} (p-w) F^{-1} \left(\frac{p-w}{p} \right) < \alpha \leq \frac{\overline{\rho}}{\overline{\rho} - \left(1 - \frac{w}{p} \right)} \cdot \frac{w}{p} (p-w) F^{-1} \left(\frac{p-w}{p} \right),$$

then $\rho_{l}^{*} \geq \rho_{h}^{*}$;

If $o < \alpha \le \frac{w}{p}(p-w)F^{-1}\left(\frac{p-w}{p}\right)$, then two scenarios are considered as follows:

(1) when
$$F\left(\frac{w}{p}F^{-1}\left(\frac{p-w}{p}\right)\right) < \beta \leq F\left(\frac{\alpha+w\overline{q}}{p\overline{\rho}}\right)$$
, we have $\rho_{l_{2}}^{*} \geq \rho_{h}^{*}$;
(2) when $F(q^{\circ}) < \beta \leq F\left(\frac{w}{p}F^{-1}\left(\frac{p-w}{p}\right)\right)$, we have $\rho_{l_{2}}^{*} \leq \rho_{h}^{*}$.

Theorem 2 demonstrates the ρ value under different combinations of α and β . It is proved that the manufacturer's willing-to-pay investment in advertising is significantly different as the degree of the retailer's risk aversion changes. If the retailer seeks higher target profit and lower downside risk, the manufacturer increases the advertising level as a signal of increasing demand to boost the retailer's confidence in the market. However, the manufacturer would not unconditionally keep on increasing the advertising expense. Suppose the retailer refuses to bear any possible loss or require unreasonable target profit, it is impossible for the manufacturer to heavily invest in advertising. In a case like this, the manufacturer would reduce the advertising budget or even stop cooperating to avoid a potential loss.

Furthermore, we find the lowest advertising level in the situation with a retailer who accepts a low profit as well as low downside risk. The second proposition in Theorem 2 supports this argument. If the retailer demands a very low target profit with low downside risk, in most cases it is trying to make a trial order or it is unable to afford large quantities (which will be proved in Theorem 3). The relatively lower downside risk indicates that the retailer would rather gain less than be exposed to risk. As a result, the manufacturer would also save advertising money due to the small scale and insufficient capability.

The ultimate winner for the manufacturer's advertising support has a moderate degree of risk aversion. There are two combinative forms of target profit and downside risk, namely a high target profit with any feasible downside risk and a lower target profit with higher downside risk. Under these two conditions, the manufacturer would make more effort on advertising compared to the risk-neutral case. The first risk setting indicates that the retailer would spontaneously relax its risk constraint in pursuit of high returns. The target profit is a signal of potential purchasing in large volumes. So the manufacturer observes the signal and increases the advertising level to encourage ordering. The second setting is basically adopted by small and medium companies. The scale limitation forces them to operate with a low target profit. Therefore, they can either bear the relatively higher risk or avoid any possible risk. For the one who relaxes the risk constraint, the manufacturer would also pay more advertising effort to encourage the retailer to order more. On the other hand, the conservative retailer avoids bearing any risk and the manufacturer will not waste a large amount of money to help with its sales.

The Retailer's Ordering Policy

Theorem 2 indicates that the manufacturer's advertising budget is significantly affected by the retailer's target profit and downside risk. Note that a retailer with relatively low downside risk can also receive the "big investment" in advertising when its target profit is sufficiently high. So we can deduce that high target profit dominates downside risk in the manufacturer's advertising decision. The analysis above reveals that the retailer's risk aversion influences the manufacturer's advertising decision. Concurrently, the manufacturer's advertising effort also has a reverse impact on the retailer's order quantity. Hence, the retailer's ordering decision is complicated because of the influence from both its risk aversion and the manufacture's advertising effort. Theorem 3 derives some propositions in the retailer's ordering policy.

Theorem 3: The risk-averse retailer tends to order in a larger quantity when the manufacturer's advertising effort conforms to certain terms, including the following:

$$\begin{split} & \text{If } \frac{w}{p} \Big(p - w \Big) F^{-1} \left(\frac{p - w}{p} \right) < \alpha \leq \frac{\overline{\rho}}{\overline{\rho} - \left(1 - \frac{w}{p} \right)} \cdot \frac{w}{p} \Big(p - w \Big) F^{-1} \left(\frac{p - w}{p} \right) \\ & \text{and } \rho_{l_{i}}^{*} \geq \frac{\overline{\rho}^{2}}{\overline{\rho} - \left(1 - \frac{w}{p} \right)}, \text{ then } q_{l_{i}}^{*} \geq q_{h}^{*}; \\ & \text{If } o < \alpha \leq \frac{w}{p} \Big(p - w \Big) F^{-1} \left(\frac{p - w}{p} \right) \Big) \text{ and } \\ & F \left(\frac{w}{p} F^{-1} \left(\frac{p - w}{p} \right) \right) < \beta \leq F \left(\frac{\alpha + w \overline{q}}{p \overline{\rho}} \right), \\ & \text{ when } \rho_{l_{a}}^{*} \geq \frac{\overline{\rho}^{2}}{\overline{\rho} - \left(1 - \frac{w}{p} \right)}, \text{ we have } q_{l_{a}}^{*} \geq q_{h}^{*}; \\ & \text{ If } o < \alpha \leq \frac{w}{p} \Big(p - w \Big) F^{-1} \left(\frac{p - w}{p \overline{\rho}} \right) \text{ and } \\ & F \Big(q^{o} \Big) < \beta \leq F \left(\frac{w}{p} F^{-1} \left(\frac{p - w}{p} \right) \right), q_{l_{a}}^{*} \leq q_{h}^{*}. \end{split}$$

Theorem 3 challenges the conventional wisdom concerning the ordering policy of a risk-averse retailer in the newsvendor model. It is interesting that a risk-averse retailer's order quantity is not necessarily lower than that of a risk-neutral one due to the impacts of the manufacturer's advertising effort. This result sharply contrasts to the situation without manufacturer's advertising effort. In Gan, Sethi, and Yan's model (2005), the risk-averse retailer orders strictly less than the risk-neutral one with other parameters set as the same. It is also worth noting that this difference can only be found when the values of target profit and downside risk are restricted in specific regions. These regions are consistent with those in Theorem 2. Also, we find that high-target profit dominates downside risk in the manufacturer's advertising decision. When the retailer's target profit is high enough, it would order more products as long as the manufacturer provides enough advertising support. When the retailer expects low target profit, the advertising stimulation on order quantity works only if the retailer can bear higher downside risk. If it cannot relax the downside risk constraint, it should cut down the inventory level.

From theorems 2 and 3, we can conclude that greater investment in advertising can boost larger order quantities when the retailer expects high target profits. As for the retailers with low target profits, whether that advertising effort works is decided by its downside risk. On the other hand, the manufacturer also balances its advertising cost and expected revenue when facing retailers with heterogeneous degrees of risk aversion.

Manufacturer Advertising with Two Independent Retailers

Our model concerns the Stackelberg game with one manufacturer and one retailer. However, in reality, a large-scale manufacturer that leads a supply chain always collaborates with numerous distributors or retailers. To better understand the complicated supply chain network and gain managerial implications, we developed our model into a two-retailer game and carried out an elementary analysis. Suppose there are two retailers R_1 and R_2 as the monopolists of two independent markets and order from a common manufacturer M. R_1 and R_2 are both risk-averse players with different target profits and downside risks (α_1, β_1) and (α_2, β_2) , respectively. There are multiple combinations of α and β , but only the comparable pairs of (α_1, β_1) and (α_2, β_2) matter to our analysis. Without loss of generality, we assume R_1 has a higher degree of risk aversion compared to R_2 , and it can be expressed as $\alpha_1 > \alpha_2$, $\beta_1 < \beta_2$. For simplicity we let the two markets have the same distribution density f(x) and distribution function F(x).

The game sequence is similar to the single-retailer model: first the manufacturer advertises for the product in the two independent markets with effort ρ_1 and ρ_2 ; then the manufacturer wholesales products to both retailers at unit cost *c* and receives *w* for each unit, and the retailers will

72 / TRANSPORTATION JOURNALTM

resell them at price *p* per unit. The two retailers' maximizing problems are defined as follows:

$$\max_{q_{1} \geq 0} \pi_{r}^{1} = pE \min(q_{1}, \rho_{1}X) - wq_{1}$$
s.t. $P(p\min(q_{1}, \rho_{1}X) - wq_{1} \leq \alpha_{1}) \leq \beta_{1}$

$$\max_{q_{1} \geq 0} \pi_{r}^{2} = pE\min(q_{2}, \rho_{2}X) - wq_{2}$$
(P₄)

s.t.
$$P(p\min(q_2, \rho_2 X) - wq_2 \le \alpha_2) \le \beta_2$$
 (P_s)

The manufacturer's maximizing problem is:

$$\max_{\rho_1 \ge 1, \rho_2 \ge 1} \pi_m = (w - c)(q_1 + q_2) - (V(\rho_1) + V(\rho_2))$$
(P₆)

Theorem 4: If both downside risk constraints bind, the equilibrium strategy is:

$$q_{1}^{*} = \frac{p\rho_{1}^{*}F^{-1}(\beta_{1}) - \alpha_{1}}{w}, \quad V'(\rho)|_{\rho = \rho_{1}^{*}} = \frac{(w - c)p}{w}F^{-1}(\beta_{1});$$
$$q_{2}^{*} = \frac{p\rho_{2}^{*}F^{-1}(\beta_{2}) - \alpha_{2}}{w}, \quad V'(\rho)|_{\rho = \rho_{2}^{*}} = \frac{(w - c)p}{w}F^{-1}(\beta_{2}).$$

Theorem 5: When both retailers' constraints bind, the manufacturer invests higher advertising levels for the retailer with lower risk aversion that orders a larger quantity than the other with higher degree of risk aversion. The relationship of the variables is $\rho_1^* < \rho_2^*$, $q_1^* < q_2^*$.

Theorems 4 and 5 imply that the retailer's risk aversion directly impacts the manufacturer's advertising decision. The result is also in consistency with theorems 2 and 3. The manufacturer has larger strategy sets when it collaborates with multiple retailers. Although risk aversion regularly is a barrier of efficient collaboration between upstream and downstream players, the manufacturer saves money from costly publicizing activities as well. On the other hand, a risk-averse retailer may get less profit because of inventory liquidating, but it also avoids the overstock problem. In the real world, it is common that the players choose strategies according to risk considerations, and the results are not all negative effects.

Conclusion

This article is motivated by the desire to explain the manufacturer's advertising effort as a common market phenomenon that is hitherto largely neglected by supply chain researchers. We use the VaR measure to prove the impact of the retailer's risk aversion on the manufacturer's advertising decision. The two critical parameters that define the retailer's risk aversion are investigated as a supplement to illustrate the phenomenon. Concurrently, the retailer's ordering policy is also influenced by the manufacturer's advertising level. To generalize the scope of our work, we discuss the case for two independent retailers with heterogeneous degree of risk aversion. As for model development, we extended the model developed by Gan, Sethi, and Yan (2005) on the downside risk analysis and designed a Stackelberg game. The approach is a combination of financial measure and game theory.

Our main focus is the relationship between supply chain members with different risk attitudes when advertising involved. According to Gan, Sethi, and Yan (2005), the retailer's risk aversion can be classified according to the target profit and downside risk. The combination of higher target profit and lower downside risk leads to increased risk aversion. In our analysis, other matches of target profit and downside risk, especially moderate risk-aversion combinations, are investigated as a supplement to the risk-aversion study. To address this issue, we compared the equilibrium strategies derived under different risk scenarios and obtained insights into the interaction of the risk-averse retailer and the risk-neutral manufacturer.

First, there exists an upper bound for the retailer's target profit; otherwise the equilibrium strategy is unavailable. Second, the retailer's target profit and downside risk have direct influence on the manufacturer's advertising investment. In other words, the manufacturer will increase its advertising effort when the retailer has a moderate degree of risk aversion, while decreasing for a highly risk-averse one. Third, although conventional wisdom suggests that a risk-averse retailer definitely reduces its order quantity, we find that a manufacturer's advertising can effectively prevent the risk-averse retailer from downsizing inventory when its target profit and downside risk fall in specific regions. Fourth, high target profit dominates downside risk in both the manufacturer's advertising decision and the retailer's ordering decision. When target profit is low, these two decisions are determined by downside risk. Fifth, when there are two independent retailers with different degrees of risk aversion, the manufacturer would give more effort on advertising for the less risk-averse one.

Our main contribution is that this work highlights the impact of the downstream player's risk aversion on the upstream partner's decisionmaking. Distinguished from existing research on advertising toward end customers, this study stresses the power of advertising in the wholesale market where the manufacturer deals with the risk-averse retailer. We propose that in addition to being aware of the customer of the brand, the manufacturer can effectively prevent the risk-averse retailer from downsizing the order quantity through advertising. This work is an exploration into the research on promotion and risk. There are still many problems unsolved and questions unanswered, for example, the introduction of competition mechanisms and designing contracts to coordinate the supply chain and reallocate market risk. Our future research will go deeper into this topic to obtain more insights for both theory development and practical application.

Appendix

Proof of Theorem 1

If
$$o < \alpha \le \frac{\overline{\rho}}{\overline{\rho} - \left(1 - \frac{w}{p}\right)} \cdot \frac{w}{p} (p - w) F^{-1} \left(\frac{p - w}{p}\right)$$
, then we have $F(q^{\circ}) \le F\left(\frac{\alpha + w\overline{q}}{p\overline{\rho}}\right)$.

1. When $\beta \ge F\left(\frac{\alpha + w\bar{q}}{p\bar{\rho}}\right)$, the retailer's downside risk constraint does not bind; therefore the retailer's order decision is the same as that of the traditional newsvendor,

which is given by $\bar{q} = \bar{\rho}F^{-1}\left(\frac{p-w}{p}\right)$. Then we substitute \bar{q} into Eq. (2) and solve $\max_{\rho \geq 1} \pi_m$ for the manufacturer's optimal promotional effort $\bar{\rho}$, which can be simply obtained through first derivative condition. The equilibrium strategy $(\bar{\rho}, \bar{q})$ takes the form

$$V(\rho)|_{\rho=\overline{\rho}} = (w-c)F^{-1}\left(\frac{p-w}{p}\right), \ \overline{q} = \overline{\rho}F^{-1}\left(\frac{p-w}{p}\right).$$

2. When $F(q^{\circ}) < \beta < F\left(\frac{\alpha + w\overline{q}}{p\overline{\rho}}\right)$, the downside risk constraint binds and the

retailer's maximization problem becomes

$$\max_{q \ge 0, \ \rho \ge 1} \pi_r = pE\min(q, \ \rho X) - wq$$

s.t. $P(\prod_r \le \alpha) = \beta$ (P₁)

We can now derive the solution to (P3), as shown below:

$$q^{*} = \frac{p\rho F^{-1}(\beta) - \alpha}{w}$$
(7)

On substituting this equation into (P2) and solving for ρ^* , we obtain (ρ^*, q^*) as follows:

$$\mathbf{V}^{\cdot}(\rho)|_{\rho=\rho^{\star}} = \frac{(w-c)p}{w} F^{-1}(\beta), \ q^{\star} = \frac{p\rho^{\star}F^{-1}(\beta)-\alpha}{w}$$

3. When $\beta \leq F(q^{\circ})$, there is no such *q* that achieve the target profit α , making the whole problem unsolvable.

If
$$\frac{\overline{\rho}}{\overline{\rho} - \left(1 - \frac{w}{p}\right)} \cdot \frac{w}{p} (p - w) F^{-1} \left(\frac{p - w}{p}\right) < \alpha \le \overline{\rho} (p - w) F^{-1} \left(\frac{p - w}{p}\right)$$
, then
 $F \left(\frac{\alpha + w\overline{q}}{p\overline{\rho}}\right) < F(q^{\circ}) < F(\overline{q})$.

When $\beta \leq F(q^{\circ})$, it is obvious that no appropriate q matches; when $F(q^{\circ}) < \beta \leq 1$, it can be deduced that we have $\beta > F\left(\frac{\alpha + w\overline{q}}{p\overline{\rho}}\right)$, so the downside risk constraint does not bind, and we obtain the equilibrium strategy $(\overline{\rho}, \overline{q})$.

If
$$\alpha > \overline{\rho}(p-w)F^{-1}\left(\frac{p-w}{p}\right)$$
, then $F\left(\frac{\alpha+w\overline{q}}{p\overline{\rho}}\right) < F(\overline{q}) < F(q^{\circ})$. When $F(q^{\circ}) < \beta \le 1$,
 $\beta > F\left(\frac{\alpha+w\overline{q}}{p\overline{\rho}}\right)$, the optimal order quantity is \overline{q} , which contradicts the fact $\overline{q} < q^{\circ}$,

leaving our problem unsolvable.

Proof of Theorem 2

If
$$\frac{w}{p}(p-w)F^{-1}\left(\frac{p-w}{p}\right) < \alpha \leq \frac{\overline{\rho}}{\overline{\rho} - \left(1 - \frac{w}{p}\right)} \cdot \frac{w}{p}(p-w)F^{-1}\left(\frac{p-w}{p}\right)$$
, obviously

we have $\frac{w}{p}F^{-1}\left(\frac{p-w}{p}\right) < \frac{\alpha}{p-w} \le \frac{\overline{\rho}}{\overline{\rho} - \left(1 - \frac{w}{p}\right)} \cdot \frac{w}{p}F^{-1}\left(\frac{p-w}{p}\right)$, which is in equivalence

with the expression (8):

$$F\left(\frac{w}{p}F^{-1}\left(\frac{p-w}{p}\right)\right) < F(q^{\circ}) \le F\left(\frac{\alpha+w\overline{q}}{p\overline{\rho}}\right)$$
(8)

With $\beta > F(q^{\circ})$ we can deduce that $\beta > F\left(\frac{w}{p}F^{-1}\left(\frac{p-w}{p}\right)\right)$ and $F^{-1}(\beta) > \frac{w}{p}F^{-1}\left(\frac{p-w}{p}\right)$. Given that $V'(\rho)|_{\rho=\overline{\rho}} = (w-c)F^{-1}\left(\frac{p-w}{p}\right)$ and $V'(\rho)|_{\rho=\rho_{1}^{*}} = \frac{(w-c)p}{w}F^{-1}(\beta)$, we compare ρ_{1}^{*} to $\overline{\rho}$ and find that $\rho_{1}^{*} \ge \rho_{h}^{*}$.

If
$$o < \alpha \le \frac{w}{p}(p-w)F^{-1}\left(\frac{p-w}{p}\right)$$
, then $F(q^{\circ}) \le F\left(\frac{w}{p}F^{-1}\left(\frac{p-w}{p}\right)\right) < F\left(\frac{\alpha+w\overline{q}}{p\overline{\rho}}\right)$

Therefore, two ranges of β are considered with the downside risk constraint binding:

When
$$F\left(\frac{w}{p}F^{-1}\left(\frac{p-w}{p}\right)\right) < \beta \le F\left(\frac{\alpha+w\overline{q}}{p\overline{\rho}}\right)$$
, we have $V'(\rho_l^*) > V'(\rho_h^*)$ and $\rho_{l_2}^* \ge \rho_h^*$;
When $F(q^\circ) < \beta \le F\left(\frac{w}{p}F^{-1}\left(\frac{p-w}{p}\right)\right)$, we have $V'(\rho_l^*) \le V'(\rho_h^*)$ and $\rho_{l_3}^* \le \rho_h^*$.

Proof of Theorem 3

If
$$o < \alpha \le \frac{\overline{\rho}}{\overline{\rho} - \left(1 - \frac{w}{p}\right)} \cdot \frac{w}{p} (p - w) F^{-1} \left(\frac{p - w}{p}\right)$$
, then we get
$$q_{l}^{*} - q_{h}^{*} \ge \frac{p}{w} \rho_{l}^{*} F^{-1} \left(\beta\right) - \frac{\overline{\rho}^{2}}{\overline{\rho} - \left(1 - \frac{w}{p}\right)} F^{-1} \left(\frac{p - w}{p}\right);$$

Suggested in Theorem 2, there are two scenarios in which $F^{-1}(\beta) \ge \frac{w}{p}F^{-1}\left(\frac{p-w}{p}\right)$:

When
$$\frac{w}{p}(p-w)F^{-1}\left(\frac{p-w}{p}\right) < \alpha \leq \frac{\overline{\rho}}{\overline{\rho} - \left(1 - \frac{w}{p}\right)} \cdot \frac{w}{p}(p-w)F^{-1}\left(\frac{p-w}{p}\right)$$
 or

When
$$o < \alpha \le \frac{w}{p} (p - w) F^{-1} \left(\frac{p - w}{p} \right)$$
 with $F \left(\frac{w}{p} F^{-1} \left(\frac{p - w}{p} \right) \right) < \beta \le F \left(\frac{\alpha + w \overline{q}}{p \overline{\rho}} \right)$.

Therefore, for
$$\rho_l^* \ge \frac{\overline{\rho}^2}{\overline{\rho} - \left(1 - \frac{w}{p}\right)}$$
, we have $\rho_l^* F^{-1}(\beta) \ge \frac{w}{p} \frac{\overline{\rho}^2}{\overline{\rho} - \left(1 - \frac{w}{p}\right)} F^{-1}\left(\frac{p - w}{p}\right)$, and

the expression $q_l^* - q_h^* \ge 0$ is proved, leading to $q_l^* \ge q_h^*$.

If
$$\mathbf{o} < \alpha \le \frac{w}{p} \left(p - w \right) F^{-1} \left(\frac{p - w}{p} \right)$$
 and $F(q^{\circ}) < \beta \le F\left(\frac{w}{p} F^{-1} \left(\frac{p - w}{p} \right) \right)$, we directly have $F^{-1}(\beta) \le \frac{w}{p} F^{-1} \left(\frac{p - w}{p} \right)$, so that $\rho_l^* \le \rho_h^*$ and $q_l^* - q_h^* \le \frac{p}{w} \rho_l^* F^{-1}(\beta) - \overline{\rho} F^{-1} \left(\frac{p - w}{p} \right) \le \mathbf{o}$,

Therefore $q_l^* \leq q_h^*$ is proved.

Proof of Theorem 4

When both retailers' constraints binds, programming (P4) and (P5) can be rewritten as:

$$\max_{q_1 \ge 0} \pi_r^1 = p E \min(q_1, \rho_1 X) - w q_1$$

s.t. $P(p \min(q_1, \rho_1 X) - w q_1 \le \alpha_1) = \beta_1$ (P₇)

$$\max_{q_{z}\geq 0} \pi_{r}^{2} = pE\min(q_{z}, \rho_{z}X) - wq_{z}$$

s.t. $P(p\min(q_{z}, \rho_{z}X) - wq_{z} \leq \alpha_{z}) = \beta_{z}$ (P₈)

We can now derive the solution to programming (P_6) and (P_7) , as shown below:

$$q_{1}^{*} = \frac{p\rho_{1}^{*}F^{-1}(\beta_{1}) - \alpha_{1}}{w} \text{ and } q_{2}^{*} = \frac{p\rho_{2}^{*}F^{-1}(\beta_{2}) - \alpha_{2}}{w}$$

On substituting this equation into (P_6) and solving for ρ_1^* and ρ_2^* , we obtain (ρ_1^*, q_1^*) and (ρ_2^*, q_2^*) as follows:

$$\begin{aligned} q_{1}^{*} &= \frac{p\rho_{1}^{*}F^{-1}(\beta_{1}) - \alpha_{1}}{w}, \ \nabla'(\rho)|_{\rho = \rho_{1}^{*}} = \frac{(w - c)p}{w}F^{-1}(\beta_{1}); \\ q_{2}^{*} &= \frac{p\rho_{2}^{*}F^{-1}(\beta_{2}) - \alpha_{2}}{w}, \ \nabla'(\rho)|_{\rho = \rho_{2}^{*}} = \frac{(w - c)p}{w}F^{-1}(\beta_{2}). \end{aligned}$$

Proof of Theorem 5

As
$$\beta_1 < \beta_2$$
, we have $V'(\rho)|_{\rho=\rho_1^*} = \frac{(w-c)p}{w}F^{-1}(\beta_1) < \frac{(w-c)p}{w}F^{-1}(\beta_2) = V'(\rho)|_{\rho=\rho_2^*}$.

Because $V(\rho) > 0$, $V(\rho)$ is a monotonic increasing function. Therefore $\rho_1^* < \rho_2^*$ is derived.

As
$$\rho_1^* < \rho_2^*$$
, $\alpha_1 > \alpha_2$, $q_1^* = \frac{p\rho_1^*F^{-1}(\beta_1) - \alpha_1}{w} < \frac{p\rho_2^*F^{-1}(\beta_2) - \alpha_2}{w} = q_2^*$, $q_1^* < q_2^*$ is also

proved.

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