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I

C. I. Lewis was a leading figure in Harvard's Department of Philosophy when I was a graduate student there in the middle of the past century. Tall, dignified, and reserved in manner, he stood as a link with the golden age of Harvard philosophy.[1] He could be kindly and genial, but did not hesitate to express disapproval upon occasion. Although he had been unwell and was nearing the end of his career at Harvard, he continued to be admired by students, and his courses were well received. In his teaching he did not try to entertain with anecdotes or digressions, but kept to the subject at hand. He concentrated on what he found most valuable in the views of thinkers he treated, skirting the intricacies of their arguments when these did not impress him favorably. I heard him lecture to sizeable classes on Kant, on epistemology, and on social ethics, and I attended a seminar of his on ethics. In the seminar he read each time from manuscript he had been preparing, afterwards inviting questions. In his lectures, though, he talked fluently, using few notes, and often rose to a spirited pitch of enthusiasm about, say, Kant's account of apperception. Usually his lectures were held on the first floor of Emerson Hall in a modest classroom which I believe had been Royce's. The room had a small blackboard on the west wall behind a podium and pulpit-like lectern from which the lecturer rather towered over those sitting on the old-fashioned benches below. Lewis had a monocle which he fixed in his right eye when he occasionally read aloud from a text, but when he finished reading he would relax his right brow and let the monocle drop; new students could be startled by this maneuver, expecting the monocle to fall and shatter, but it was saved by the black ribbon attached to it and to a bead behind his ear.

In those days many of us who knew Lewis were inclined to regard him as the most eminent figure then active in American philosophy.[2] Over the decades since then, however, he has become less well known, and, regrettably, philosophy students of today often are not acquainted with his work. It is appropriate that there now be a review and reassessment of Lewis's standing in twentieth-century American philosophy. Murray Murphey's impressive new study of Lewis's life and thought provides this in a most welcome manner.[3] It presents clear and enlightening summaries of Lewis's various writings, and often combines these with judicious evaluation and criticism.

In his Chapter 3, however, which deals with Lewis's work in logic, Murphey does less in the way of offering evaluative comment than he does in his later chapters on Lewis's work in epistemology and ethics. It is true that in several places in Chapter 3 Murphey does say that particular views of Lewis's are less than clear, and this is criticism of a gentle sort. Yet the overall tone of Chapter 3 is descriptive rather than critical, and this is especially noticeable when Murphey discusses Lewis's attack on Russell's idea of material implication. That attack surely was the most heatedly controversial part of Lewis's work in logic. In dealing with it, Murphey clearly and helpfully sets forth what Lewis said about implication, but does not offer much by way of defense or criticism of it. Because of this non-judgmental presentation, some readers of Murphey's book might carry away the impression that Lewis won out over Russell in their clash concerning implication. That impression would be misleading, I believe. I want to try to oppose such an impression by spelling out why, as I see it, neither Lewis nor Russell was a winner in this controversy, each being partly in the right but partly in the wrong. Although the controversy is no longer a lively one, the matter deserves the attention of those interested in Lewis, since his views about implication provided the stimulus for much of his early work.

II

The idea of material implication had been introduced near the beginning of *Principia Mathematica*.[4] Whitehead shared with Russell responsibility for the whole of that vast book, but the early sections seem to have been drafted mainly by Russell.[5] In Chapter I of the Introduction Russell dealt with negation, disjunction, and conjunction, and assigned them symbols. He then introduced a further truth-functional connective (he called it a relation) which he proposed to express via the horseshoe symbol, as in "$p \supset q$." He defined this as equivalent to "not*p* or *q*." He called this connective "material implication," and said it is to be read, "*p* implies *q*."

What motivated Russell to choose "implies" as his reading of the horseshoe symbol? The explanation he gave in Chapter I of the *Principia* was that if "not*p* or *q*" is true and *p* is also true, then *q* must be true; thus "not*p* or *q*" enables us to deduce *q* from *p*, and in this sense it and its horseshoe version mean that *p* implies *q*. Here it seems that Russell was regarding this specific truth-functional compound as unique in its ability to make possible the

inferring of q from p; apparently he was thinking that proofs in general have the form "$p, p \supset q$, so q." Also we should note that in this passage Russell was abiding by the familiar verbal usage of logicians who identify the deducibility of q from p with the implication of q by p.

Lewis strongly disapproved of Russell's material implication, as Murphey explains in his Chapter 3. Lewis interpreted Russell as advocating a general principle to the effect that, for q to be deducible from p, it is sufficient that either p be false or q be true. The passage just referred to in Chapter I of the *Principia* does indeed suggest that Russell at that point was thinking along those lines. Such a principle concerning deducibility is highly questionable, of course. One can object to it by pointing out that even if it is known to be false that pigs fly, this is quite insufficient to establish that every proposition is deducible from the proposition that pigs fly. Some of us would want to say that the general principle in question is flatly false, and Lewis sometimes seems willing to say this, though on other occasions he says more mildly, in a pragmatic spirit, that a system of logic which accepts this principle is not useful to us.

Lewis, in some of his early writing, accused the logic of material implication of carrying with it a monistic metaphysical theory having various outrageous theses, such as that true propositions all are necessarily true. Here presumably he was thinking that necessarily true propositions, and only they, are implied by every proposition; and that, since in *Principia* any true proposition is implied by every proposition, it follows that the logic of *Principia* conflates truth with necessity. Lewis's distaste for such a metaphysics seems to have heightened his disapproval of material implication.

Lewis went on to urge that in *Principia Mathematica* all the proposed deductions of theorems are unsound, and all the theorems remain unproved, since an illogical conception of deducibility has been employed throughout. This, of course, is a very severe criticism of the work of Russell and Whitehead. Lewis seems to have been agreeing with Russell that all proofs in *Principia* have the schematic form "$p, p \supset q$, so q," and from this Lewis seems to have concluded that all those proofs essentially depend on material implication, the inadequacy of which undermines their soundness.

Endeavoring to restore soundness to mathematical logic, Lewis introduced his relation of strict implication, where "p strictly implies q" is to mean that q is genuinely deducible from p. He devised symbols for strict implication and associated notions, framed postulates to govern them, and combined these with postulates from the *Principia* to form a blended system. By putting strict implication directly into the postulates and theorems of the system, Lewis proposed to correct what he saw as the unsoundness of *Principia*.

III

There are several weaknesses, however, in Lewis's criticisms of what Russell had said about material implication.

Russell's incautious presentation had indeed suggested that from any false proposition any proposition whatever is deducible, and that from any proposition whatever any true proposition is deducible. One can understand how Lewis became incensed at such theses. Yet, though in some of his moments Russell did appear to endorse these theses, he at any rate never put them into practice as he constructed proofs for *Principia*. Never in those proofs do there occur steps where a conclusion is drawn merely on the grounds that its premises are not all true, or merely on the grounds that the conclusion itself is true.

Indeed, such absurd steps would violate the very notion of what each step in a direct deductive proof is supposed to achieve. Each such step is supposed to start from premises all of which are regarded as true, and is supposed to arrive at a conclusion whose truth was not initially presupposed.[6] Russell was too good a logician to put into *Principia* absurd steps that violate these requirements, nor would Whitehead have tolerated their presence. *Principia*'s proofs indeed achieve much success in validly deducing conclusions from premises.

Thus Lewis was overstating his case when he said that the proofs of *Principia* do not prove its theorems. What Lewis might better have said is that, had Russell whole-heartedly and systematically endorsed the objectionable theses mentioned above, he would have felt free to introduce into his proofs absurd steps that would have vitiated the reasoning, and the whole system could have become unsound. In fact, though, Russell avoided falling so deeply into error.

Lewis's distaste for the horseshoe symbol, when it is read as "implies," perhaps led him to think that the pervasive presence of this symbol in *Principia* carries with it the taint of the absurd theses mentioned above, and thereby saps the soundness of its proofs. Many of the steps in these proofs do have the schematic form: "p, $p \supset q$, so q" (though by no means all of them do, despite what Russell and Lewis both seem sometimes to have supposed). This schematic form, though, is just an elementary form of valid truth-functional inference (were anyone to doubt that it is valid, the truth-table method offers a way of verifying its validity). Thus, the validity of this form of inference need not be regarded as suspect. When we make inferences of this form, we do not have to be endorsing any absurd theses about implication.

If the system of *Principia* in its original form had been inconsistent because of some fault in its truth-functional postulates, the way to make it consistent would have been to alter those truth-functional postulates. Adding modal symbols and postulates to govern them would not be the effective way to cope with such a difficulty.

Moreover, introducing into the basic language of logic a symbol for strict implication is not the best way to straighten out Russell's confusing treatment of implication. It would be better to employ a meta-language, and state in it the conditions under which propositions in the object language are deducible from one another. At the time when Lewis was writing about

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logic, philosophers were not yet familiar with the distinction between object-language and meta-language, so of course he is not to be criticized for not having employed it, helpful though it would have been.

The kindred distinction between using and mentioning expressions had, however, already been drawn and forcefully emphasized by Frege, especially in his *Foundations of Arithmetic*.[7] This book was not translated into English until later, of course, but if Lewis had made use of it he could have formulated his objections against Russell in a better-focussed manner. Here what is needed is to distinguish between two different types of logical grammar. When we write the horseshoe symbol between two propositions we use them to form a compound proposition that does not mention its components. In contrast, when we say that one proposition implies another, we mention each in order to attribute a relationship to them. Lewis, however, treats the horseshoe connective and strict implication as though they were on the same grammatical level, and even speaks of material implication as "a form of inference," when it is merely a truth-functional connective. This makes his presentation less clear than it might have been.

Not only was Lewis overstating his case when he said that the proofs of *Principia* are unsound, but also he was doing so when he suggested that Russell's treatment of material implication involved commitment to a monistic metaphysics in which all true propositions are necessarily true. During this same era, Russell was advocating his doctrine of Logical Atomism, which is diametrically opposed to monism. Lewis might better have said merely that Russell's account of implication, under one extreme interpretation, could have been regarded as leading toward monism.

IV

Lewis's criticisms of Russell's treatment of material implication had their weaknesses, but Russell's position certainly did deserve to be criticized.

Russell's explanation in *Principia* of why he thought it appropriate to read "not*p* or *q*" as "*p* implies *q*" is unsatisfactory. It misleadingly treats "not*p* or *q*" as though this formula were uniquely qualified to enable the deduction of *q* from *p*. This is not so, as there are any number of other truth-functional formulas which are equally effective at enabling valid deduction of *q* from *p*; for example, "not(*p* & not*q*)" and "*p* if & only if *q*." Russell did not want to read each of these as "*p* implies *q*," but his explanation provides no better reason for assigning this reading to "not*p* or *q*" than to these others.

Russell sought to defend himself against criticism of material implication by claiming that critics were missing the point. F. H. Bradley had written to Russell in 1910 on this subject, apparently making criticisms rather like those that Lewis made a little later. To Bradley, Russell replied, "I use the word 'implication' in a special technical sense which does not carry with it the consequences you indicate. I say that one proposition 'implies' another whenever the first is false or the second true (not excluding both). I do not

pretend that this is the usual meaning of the word, but it is a relation for which I need a name, and no other name occurred to me."[8]

Here one notes that, in replying to Bradley, Russell had shifted his ground. The inadequate rationale he had offered in *Principia* to support reading the horseshoe symbol as "implies" went unmentioned, and he retreated to the claim that he had merely made an arbitrary choice of a verb to correlate with this symbol. Later, when he replied to Lewis's criticism,[9] Russell offered essentially this same defense, and reiterated the claim that his critics had failed to grasp that the controversy is merely verbal.

Such a defense is not adequate, though. An author may deserve to be criticized for introducing a novel usage which creates confusion because it is needlessly at odds with already existing usage. Rather than reading the horseshoe symbol as "implies," Russell would have done better to read it as "only if." Possibly that reading would also have generated a few complaints, but they would have been far milder than the complaints generated by the reading Russell chose, which was so sharply at odds with a usage that had become familiar to logicians. Moreover, Russell's insistence on reading the horseshoe as "implies," and his unwillingness explicitly to retract the bad explanation he had initially given of why he chose this reading, inevitably stir suspicion that he had not merely made an arbitrary verbal choice, but that he himself was far from clear about how implication, as a relation between propositions, differs from the truth-functional conditional, as a propositional connective.

V

In view of these various difficulties in which Russell became entangled, and these various problems with Lewis's criticism of Russell's position, I would conclude that in this controversy neither was victorious over the other. Each was right in some respects, and wrong in other respects. To say this is not to denigrate their intelligence and energy, or their distinction as thinkers. Even if the matter they were contesting may now look simple in distant hindsight, it is well to remember that in their time it was a recalcitrant tangle, very difficult to straighten out. Those early years of twentieth-century logic were richly fruitful, yet most logicians had not yet seen how to avoid inadequate formulations of their fundamental concepts.

Although there were shortcomings in Lewis's attack on material implication, we can be most grateful that his thinking about this topic led him on to initiate the systematic study of symbolic modal logic. Modal logic has not come to play as central a role in mathematical logic as Lewis thought it would have, but the systems of modal logic Lewis constructed have turned out to be challenging, with properties very worthy of study by able logicians.[10] This is Lewis's legacy as regards implication, and it is a valuable one.

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NOTES

1. Victor Lowe says there were two golden ages of Harvard philosophy; if so, Lewis spanned both. Victor Lowe, *Alfred North Whitehead: The Man and His Work*, vol. II (Baltimore: Johns Hopkins University Press, 1998), p. 149.

2. Victor Lowe, *loc. cit.*, referring to a somewhat earlier period, offers the opinion that in the English-speaking world there was "no one who was quite Lewis's equal in analytic philosophy."

3. Murray G. Murphey, *C. I. Lewis: The Last Great Pragmatist* (New York: SUNY Press, 2005).

4. Alfred North Whitehead and Bertrand Russell, *Principia Mathematica*, vol. I (Cambridge: Cambridge University Press, 1910), p. 7.

5. Bertrand Russell, *My Philosophical Development* (London: Allen & Unwin, 1959), p. 74.

6. Russell shows strong recognition of these points in his *Introduction to Mathematical Philosophy* (London: Allen & Unwin, 1919), p. 153.

7. Gottlob Frege, *Grundlagen der Arithmetik* (Breslau: Wilhelm Koebner, 1884).

8. *Russell's Logical and Philosophical Papers, 1909–13*, John G. Slater, ed. (London: Routledge, 1992), p. 350.

9. Bertrand Russell, *Introduction to Mathematical Philosophy, loc. cit.*

10. Perhaps especially noteworthy is the work of Saul Kripke on modal logic. In his "Semantical Analysis of Modal Logics I," *Zeitschrift für mathematische Logic und Grundlagen der Mathematik* 9 (1963), pp. 67–96, and in other papers, Kripke provided a semantics for modal logic. Hintikka wrote that through the work of Kripke and others "a fascinating new set of methods and ideas was thus made available for philosophical studies" (Jaakko Hintikka, *Models for Modalities,* Dordrecht: D. Reidel, 1969, p. v).